PHYSICAL REVIEW E VOLUME 60, NUMBER 5 NOVEMBER 1999

Landau damping of an electron plasma wave in a plasma with modulated density

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(Received 21 April 1999)

The collisionless Landau damping is analyzed for a homogeneous plasma with a sinusoidal density perturbation. The exact damping values are obtained by numerically solving the kinetic dispersion relation of the electron plasma mode. Quantitative results are provided on the influence of the amplitude and the wave vector of the density modulation. Within a simplified fluid framework, a fit is given to these data.
[S1063-651X(99)51511-0]

PACS number(s): 52.40.Nk, 52.35.Nx, 52.60.+h

In the context of inertial confinement fusion (ICF), numerous works have reported over the years on Raman scattering, which is able to amplify an electron plasma wave (EPW) and a scattered electromagnetic wave (EMW). The first analytical predictions of its amplification mentioned gain lengths smaller than 10 μm, whereas the latest experimental observations of substantial backscatter (say 1% fraction of the incident energy) exhibit gain lengths of a few hundredths of a μm [1]. The usual predictions assume the electron velocity and the density distribution to be Maxwellian and homogeneous, respectively. Various papers have tried to explain the difference between theory and experiments by invoking secondary instabilities such as the Langmuir decay instability (LDI) of the EPW [2,3], kinetic effects associated with non-Maxwellian electron velocity distributions [4], or the presence of magnetic fields [5]. The relativistic motion of the electrons could also be invoked but, even in high intensity regions of the laser focal spot, the electron quiver velocity induced by the laser electric field usually stays below 10% of the velocity c of light in vacuum.

Here, we concentrate on one of the background actors of the plasma collective behavior, namely, the EPW Landau damping, which needs to be considered when the electron density is modulated and/or the velocity distribution is non-Maxwellian. The latter corresponds to the high-intensity limit in ICF plasmas whenever the electron heating by inverse bremsstrahlung is faster than the electron thermalization [6]. We limit our study to a one-dimensional plasma, thereby disregarding any refraction processes. We assume that the plasma density is perturbed by a static ion-acoustic wave (IAW; the influence of a growing IAW is outside the scope of this paper), with a sinusoidal pattern: $n_e(x) = n_{eo}[1+\varepsilon \cos(k_s x)]$, where $\varepsilon = \delta n_e / n_{eo}$ denotes the amplitude of the density modulation and k_s is the IAW wave number. The coupling between an EPW and an IAW, with respective wave numbers k and k_s, is known to lead to the generation of EPW harmonics with wave number $k_m = k + m k_s$ (m is an integer). The number of intense harmonics, denoted m_h, is roughly given by the relation $m_h = \sqrt{\varepsilon/3}(1/k_s \lambda_{De})$, where λ_{De} is the Debye shielding length [7]. The larger this number, the stronger the Landau damping of the shorter wavelength harmonics [8,3]. The relevant parameters of the problem are the density modulation ε and the normalized wave vectors $\kappa \equiv k\lambda_{De}$ and $\kappa_s \equiv k_s \lambda_{De}$. In our first step, we shall assume that the IAW is produced by the beating of an incident laser wave (wave-vector k_0) with a backscattered wave, driven by stimulated Brillouin backscattering (SBBS, the decay of the incident laser wave into an EMW and an IAW) or by reflection; the IAW wave vector is then $2k_0$. The kinetic dispersion relation in a modulated plasma was originally given by Barr and Chen [8],

$$\left(1+\frac{1}{\chi_e(k,\omega)}\right)\hat{E}(k,\omega) = -\frac{\varepsilon}{2}[\hat{E}(k+k_s,\omega)+\hat{E}(k-k_s,\omega)], \tag{1}$$

where $\hat{E}$ is the time-space Fourier transform of the EPW electric field, (ω,k) is the pair (radial frequency, wave vector) associated with the EPW's solutions of this dispersion relation, and χ_e is the electron susceptibility given by

$$\chi_e(k,\omega) = \frac{\omega_{pe}^2}{k^2 n_{eo}} \int_{-\infty}^{+\infty} \frac{\partial_v f_o}{\omega - kv} dv. \tag{2}$$

ω_{pe} is the background electron plasma frequency and f_0 is the initial velocity distribution. Following Ref. [6,9,10,4], f_0 can be cast into the general expression

$$f_0(v) = C(n)\frac{n_{eo}}{v_{Te}^3} e^{-(|v|/\alpha_e v_{Te})^n}, \tag{3}$$

where v_{Te} is the thermal velocity, $\alpha_e = \sqrt{3\Gamma(3/n)/\Gamma(5/n)}$ (Γ is the gamma function) and $C(n)$ is the normalization factor $n/[4\pi\alpha_e^3\Gamma(3/n)]$. In laser-plasma interaction, the exponent can be increased from 2 to 5 by increasing the laser intensity and consequently the heating rate. The EPW frequency and the Landau damping have been recently analyzed as a function of the exponent n in Ref. [4].

Equation (1) is solved using a Newton-Raphson algorithm that is well adapted to finding the zeros of nonlinear equations [11]. For calculating χ_e, we have chosen a general method based on the sampling theorem [11] that can deal with any hyper-Gaussian velocity distribution,

$$g_n(t) \equiv e^{-|t|^n} = \sum_{m=-\infty}^{+\infty} e^{-|t_m|^n} \sin c\left[\frac{\pi}{h}(t-t_m)\right] + \mathrm{err}(t) \tag{4}$$

where $\sin c(x) \equiv \sin(x)/x$, $t_m = mh$, h is the sampling time step, and $\mathrm{err}(t)$ is the error, which can be disregarded if the Fourier transform $G_n(\omega) \equiv FT(g_n)$ is small enough for $|\omega| \geqslant 1/h$. In our case, g_n and G_n decrease sufficiently for large t and ω that we can reasonably truncate the series in Eq. (4). After some algebra and defining $z = \omega/k\lambda_{De}$, Eq. (2) becomes [for $\mathrm{Im}(z) > 0$]

$$\chi_e(k,\omega) = \frac{2\pi\alpha_e C(n)}{(k\lambda_{De})^2}\left[\frac{2}{n}\Gamma\left(\frac{1}{n}\right) + \frac{z}{\alpha_e} Z_n\left(\frac{z}{\alpha_e}\right)\right], \quad (5)$$

with

$$Z_n(z) = \sum_{m=-N}^{+N} h e^{-|t_m|^n} \frac{1-(-1)^m e^{-i(\pi z/h)}}{t_m - z}. \quad (6)$$

For the half-space $\mathrm{Im}(z) < 0$, we use $Z_n(z) = [Z_n(z^*)]^* + 2i\pi e^{-z^n}$ (where * denotes the complex conjugate). For $n=2$ (respectively 5), we use $(N,h) = (25,0.25)$ [respectively (100,0.02)]. To choose the pair (N,h), we compared the exact value of the integral $\int_0^\infty e^{-x^n} dx = (1/n)\Gamma(1/n)$ to the result obtained from the integration of the sampled function. We can assert that the relative error is below 10^{-15} (respectively 10^{-9}), for $n=2$ (respectively 5). For $n=2$, the results of our method have been compared to those given by another routine that uses an expansion of χ_e to order z^5 for $|z| < 1$, a Gaussian integration for $|z|$ in the range [1,5] and an asymptotic expansion for $|z| > 5$ [12]: a 0.1% difference was found. For $n=5$, we have compared our results to those in Ref. [4]; the latter are well-reproduced to within a few %. Our method requires less computer time: to compute χ_e using Eq. (5) for one pair (ω,k) and for typical ICF parameters ($\kappa = 0.34$) requires only 0.15 ms (respectively 1 ms) for $n=2$ (respectively 5) on a workstation ULTRA Sparc II.

In addition to the kinetic model and to increase our understanding, we have developed a fluid model in which the susceptibility is given by the relation $1 + 1/\chi_e(k,\omega) = -\omega^2/\omega_{pe}^2 + 1 + 3\kappa^2 - 2i\gamma_L(\kappa,0)/\omega_{pe}$, where $\gamma_L(\kappa,0)$ is a fit of Landau damping in a homogeneous plasma (0 stands for $\varepsilon = 0$). For $n=2$, the fit expression is obtained from Ref. [12] as

$$\frac{\gamma_L(\kappa,0)}{\omega_{pe}} = \sqrt{\frac{\pi}{8}}\frac{1}{\kappa^3} e^{-(3/2)-1/2\kappa^2}[1 + (-0.5\kappa^{0.2} + 9\kappa^{13/3}) \times (1 - e^{-(4.78\kappa)^5})]. \quad (7)$$

This fit is within a 5% error of the exact solution of the numerical solution for $\kappa \in [0,10]$. The fit appears to be the product of the usual Landau damping expression and a correction term (in brackets) that converges to one for $\kappa \to 0$. In the range [0.2–0.7], γ_L can be fitted by a simple power law $6\kappa^6$, which can be used for simulating wave couplings with a simple difference operator in real space. The κ range, where γ_L exceeds $10^{-4}\omega_{pe}$ and increases strongly with κ, is relevant to instability thresholds. Outside this range, this power law predicts too large a damping. For $n \in]2,5]$, we have defined a fit as the sum of the first 30 Chebyshev polynomials; the error is less than 10% at maximum and essentially originates from the regions where the slope of $\gamma_L(\kappa,0)$ strongly varies.

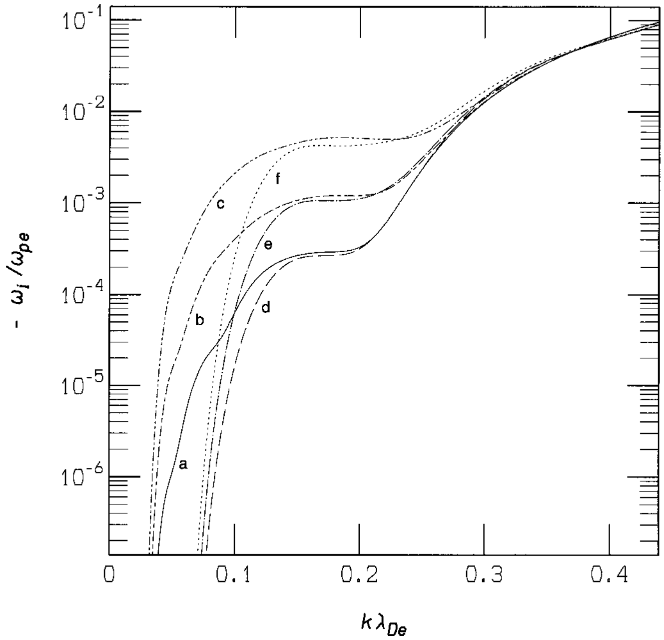

FIG. 1. EPW damping (normalized by the plasma frequency: ω_{pe}) vs EPW wave vector (normalized by the Debye shielding length: λ_{De}), for a Maxwellian velocity distribution on which a sinusoidal density modulation is superimposed with amplitude ε and wave number $k_s = 1.2k$. Numerical solution obtained with seven harmonics coupling for ε=(a) 5%, (b) 10%, and (c) 20%. The fit (with only the harmonics 0,+1) is plotted for ε= (d) 5%, (e) 10%, and (f) 20%.

Conditions for typical ICF conditions are electron temperature $T_e = 1$ keV, $\varepsilon = 0.1$, and a wave number k_s, which is given by $k_s = 2(\omega_0/c)\sqrt{1 - n_{e0}/n_c}$ for SBBS in a electron density $n_{e0}/n_c = 0.1$. (ω_0 denotes the radial frequency of the incident laser wave and n_c the critical density beyond which the laser light cannot propagate.) Under such conditions, only a few harmonics are strongly coupled ($m_h \approx 0.7$). Therefore, truncating Eq. (1) after three harmonics, −1, 0 and +1, allows us to find a correct fit to the kinetic damping $\gamma_L(\kappa,\varepsilon)$ in a modulated plasma, with the expression

$$\gamma_L(\kappa,\varepsilon) \approx \gamma_L(\kappa,0) + A_1(\kappa,\kappa_s,\varepsilon)\gamma_L(\kappa+\kappa_s,0) + A_1(\kappa,-\kappa_s,\varepsilon)\gamma_L(\kappa-\kappa_s,0). \quad (8)$$

The parameter $A_1(\kappa,\kappa_s,\varepsilon) = (\varepsilon/2)^2/[3(\kappa+\kappa_s)^2 - 3\kappa^2]^2$ denotes the coupling strength for the first harmonics [13]. Since the ratio $A_1(\kappa,\kappa_s,\varepsilon)\gamma_L(\kappa+\kappa_s,0)/A_1(\kappa,-\kappa_s,\varepsilon) \times \gamma_L(\kappa-\kappa_s,0)$ is much larger than 1, the term $A_1(\kappa, -\kappa_s,\varepsilon)$ can be disregarded. Equation (8) is not valid when $\gamma_L(\kappa+\kappa_s,0)$ vanishes (see Figs. 1 and 2).

In Figs. 1 and 2, the influence of the modulation is clearly observed as a shoulder added to the left-hand side of the region, where the Landau damping drops dramatically. Consequently, three regimes can be identified when decreasing κ: (i) for high κ values, the damping is unperturbed by the modulation; (ii) for intermediate κ values, the damping is

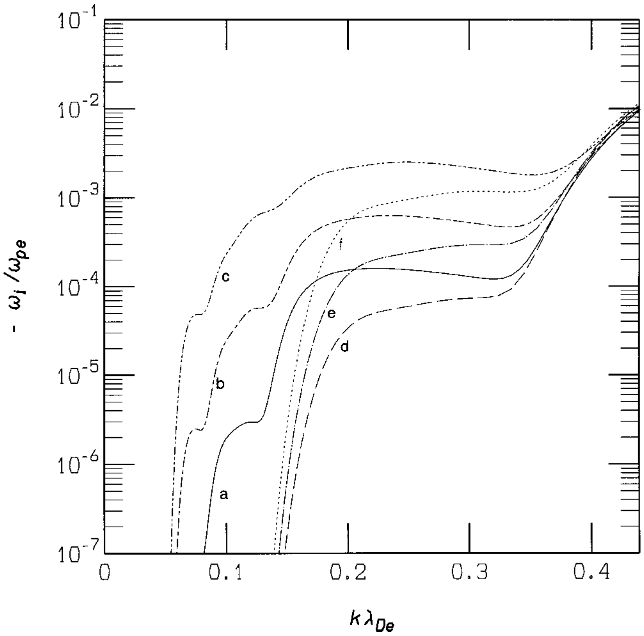

FIG. 2. EPW damping (normalized by the plasma frequency: ω_{pe}) vs EPW wave vector (normalized by the Debye shielding length: λ_{De}) for a hyper-Gaussian velocity distribution. Same conditions as in Fig. 1.

constant, since it arises from a balance between the increase of $A_1(\kappa,\kappa_s,\varepsilon)$ and the decrease of $\gamma_L(\kappa+\kappa_s,0)$; (iii) for small κ values, where the damping $\gamma_L(\kappa+\kappa_s,0)$ decreases exponentially, the global EPW damping drops. The limit between regimes (i) and (ii) is roughly given by the equality

$$\frac{\gamma_L(\kappa,0)}{\gamma_L(\kappa+\kappa_s,0)}=A_1(\kappa,\kappa_s,\varepsilon). \tag{9}$$

The coupling parameter is weak; e.g., for $\varepsilon\leqslant 0.3$ and $\kappa\approx 0.3$, we find $A_1(\kappa,\kappa_s,\varepsilon)\leqslant 0.02$. Consequently, the influence of the density modulation becomes evident when the damping ratio in Eq. (9) is larger than 10^4. Therefore, for any modulation ε, the boundary between the regimes (ii) and (iii) is located where $\gamma_L(\kappa,0)$ decreases exponentially, at $n=2$ (respectively 5) and $\kappa\in[0.2,0.3]$ (respectively $[0.35,0.45]$, see Ref. [4]). The bumps observed at small κ values are due to the excitation of higher wave vectors. They are characterized by a weak coupling strength $A_m(\kappa,\kappa_s,\varepsilon)\sim(\varepsilon/2)^{2m}/[3(\kappa+m\kappa_s)^2-3\kappa^2]^2$. Equation (7) provides a good qualitative fit of the first shoulder and is sufficient to give a crude approximation at low κ.

In Figs. 3 and 4, we investigate the influence of κ_s based on Eq. (8) for $n=2$ and $n=5$. We have used a large range of modulation wave vectors $\kappa_s=m\kappa, m\in[0.1,10]$. The choice $m=2$ corresponds to a LDI-driven IAW. High (respectively low) values of m would correspond to Raman scatter driven by a 10 (respectively 1)-μm laser in a plasma modulated by a 1 (respectively 10)-μm laser-driven SBBS. When $\kappa\gg\kappa_s$, the coupling is so efficient that the shoulder appears at the large value κ. Nevertheless, since $\gamma_L(\kappa+\kappa_s,0)$ is close to $\gamma_L(\kappa,0)$, the damping is only slightly different from the no-modulation case. When $\kappa\ll\kappa_s$, the damping is given by the asymptotic expression $\gamma_L(\kappa,0)+\varepsilon^2\gamma_L(\kappa_s,0)/36\kappa_s^4$, which has a maximum in regime (ii) for $\kappa_s\approx 0.3$. This expression is

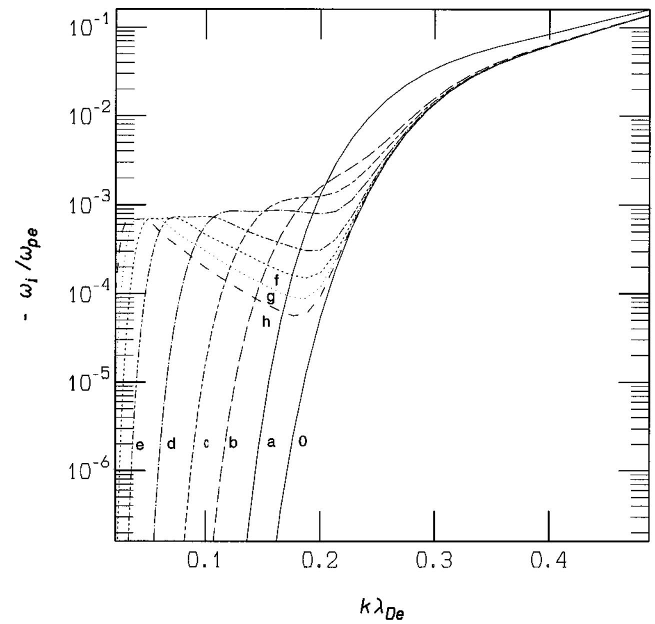

FIG. 3. EPW damping (normalized by the plasma frequency: ω_{pe}) vs EPW wave vector (normalized by the Debye shielding length: λ_{De}) for a Maxwellian velocity distribution on which a sinusoidal density modulation is superimposed with amplitude $\varepsilon=10\%$ and wave number $\kappa_s=$ (a) 0.1κ, (b) 0.5κ, (c) κ, (d) 2κ, (e) 4κ, (f) 6κ, (g) 8κ, and (h) 10κ. The solid line labeled 0 corresponds to the unmodulated case ($\varepsilon=0\%$). Numerical solution with seven harmonics.

valid for $\kappa_s\geqslant 4\kappa$, as seen in Figs. 3 and 4. In the intermediate case, $A_1(\kappa,\kappa_s,\varepsilon)$ is almost constant until $\gamma_L(\kappa+\kappa_s,0)$ drops, when $\kappa+\kappa_s\approx 0.3$. We then conclude that a density modulation, created either by a turbulence process or by the steepening of an IAW-driven SBBS, with wavelength equal to a fraction of the EPW wavelength, might result in nonzero

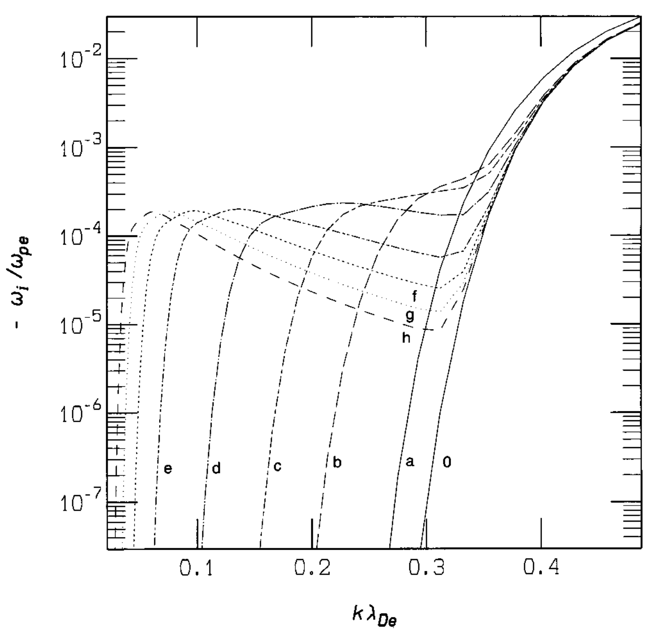

FIG. 4. EPW damping (normalized by the plasma frequency: ω_{pe}) vs EPW wave vector (normalized by the Debye shielding length: λ_{De}) for a hyper-Gaussian velocity distribution. Same conditions as in Fig. 3.

kinetic damping for any small κ. This can be shown by extending Eq. (8) to any weakly turbulent density profile $n_e(x)=n_{eo}[1+\Sigma_{p=1}^{N}\varepsilon_p\cos(k_{sp}x)]$:

$$\gamma_L(\kappa,\langle\varepsilon\rangle)\approx\gamma_L(\kappa,0)+\sum_{p=1}^{N}[A_1(\kappa,\kappa_{sp},\varepsilon_p)\gamma_L(\kappa+\kappa_{sp},0)+A_1(\kappa,-\kappa_{sp},\varepsilon_p)\gamma_L(\kappa-\kappa_{sp},0)], \tag{10}$$

where $\langle\varepsilon\rangle$ denotes the average modulation. In this new context, the damping rate exhibits two behaviors: the usual Landau damping above $\kappa_s\geqslant 0.3$ and, at lower κ, an anomalous damping that depends on the statistics of $(\varepsilon_p,\kappa_{sp})$.

In conclusion, the behavior of the kinetic damping for an EPW has been investigated under conditions where both hyper-Gaussian electron distributions and short-wavelength density modulations are expected to occur. Regimes of enhanced Landau damping have been identified as a function of the density modulation and the products $k\lambda_{De}$ and $k_s\lambda_{De}$. A static density modulation tends to keep the kinetic damping at large values in the wave-vector region where Landau damping is usually extremely low, even for hyper-Gaussian distributions. An analytic fit, calculated from a fluid model, reproduces the results well. By comparing to experiments in not too hot plasmas ($T_e=1$ keV, experiment A in Refs. [14,15]), our results clearly reproduce the experimental observation of the SRS quenching by SBS seeding. Indeed, by assuming the driven SBS-IAW amplitude to saturate at $\varepsilon=0.2$, as noticed in Ref. [15] and as would be inferred by the ion trapping mechanism [16], the damping rate of the EPW at the experimental density $n_e=0.13$ is largely increased by the density modulation, $-4.10^{-3}\omega_{pe}$ (as would be given by a homogeneous plasma at $k\lambda_{De}=0.26$) instead of $-4.10^{-6}\omega_{pe}$ in the unmodulated case ($k\lambda_{De}=0.18$). In hotter plasmas ($T_e=3$ keV) as created on NOVA laser, we have also determined the impact of the modulation by estimating the gain factors G with the usual convective highly damped EPW model. We have extended the usual expression to modulated plasmas and have found that, for a 0.25 mm long flat-density plasma, the gain is still in the linear regime ($G<20$) even for $n_e=0.2n_c$, contrasting with the no-modulation case where the gain is in excess of 20 for densities as low as $0.16n_c$ to reach 30 at $0.2n_c$. More interesting results on gains will be reported in a future publication.

One of the authors (G.B.) wishes to thank B. Afeyan for stimulating discussions.

[1] B. Mc Gowan *et al.*, Phys. Plasmas **3**, 2029 (1996); J.C. Fernandez *et al.*, Phys. Rev. Lett. **77**, 2702 (1996).

[2] B. Bezzerides, D.F. Dubois, and H.A. Rose, Phys. Rev. Lett. **70**, 2569 (1993); T. Kolber, W. Rozmus, and V.T. Tikhonchuk, Phys. Fluids B **5**, 138 (1993).

[3] G. Bonnaud, D. Pesme, and R. Pellat, Phys. Fluids B **2**, 1618 (1990).

[4] B. Afeyan, A. Chou, J.P. Matte, R.P.J. Town, and W.J. Kruer, Phys. Rev. Lett. **80**, 2322 (1998).

[5] D. Mourenas, Phys. Plasmas **5**, 243 (1998).

[6] B.C. Langdon, Phys. Rev. Lett. **44**, 575 (1980).

[7] P.K. Kaw, A.T. Lin, and J.M. Dawson, Phys. Fluids **16**, 1967 (1973).

[8] H.C. Barr and F.F. Chen, Phys. Fluids **30**, 1180 (1987).

[9] C.T. Dum, Phys. Fluids **21**, 945 (1978).

[10] J.P. Matte *et al.*, Plasma Phys. Controlled Fusion **30**, 1665 (1988).

[11] W.H. Press, S.A. Teukolsky, W.T. Vettering, and B.P. Flannery, *Numerical Recipes in Fortran*, 2nd ed. (Cambrige University Press, 1992), pp. 376–381.

[12] See AIP Document No. PAPS-PFBPE-02-1608-208 for 208 pages of Etude Théorique du Comportement Electrodynamique du Plasma by Guy Bonnaud, CEA Report No. CEA R-5477 (1989). This document may be retrieved via the EPAPS homepage (http://www.aip.org/pubservs/epaps.html) or from ftp.aip.org in the directory /epaps/. See the EPAPS homepage for more information. The susceptibility calculation uses the method in W.D. Jones, H.J. Doucet, and J.M. Buzzi, *An Introduction to the Linear Theories and Methods of Electrostatic Waves in Plasmas* (Plenum Press, London, 1985).

[13] M. Curtet and G. Bonnaud, C. R. Acad. Sci., Ser. I: Math. **327**, 799 (1999).

[14] H.A. Baldis, D.M. Villeneuve, C. Labaune, D. Pesme, W. Rozmus, W.L. Kruer, and P.E. Young, Phys. Fluids B **3**(8), 2341 (1991).

[15] C.J. Walsh, D.M. Villeneuve, and H.A. Baldis, Phys. Rev. Lett. **53**, 1445 (1984).

[16] J.M. Dawson, W.L. Kruer, and B. Rosen, *Dynamics of Ionized Gases*, edited by M. Lighthill, I. Imai, and H. Saro (University of Tokyo Press, Tokyo, 1973), p. 73.